Experimental Exam

Problem. Finding the band gap of germanium.

Introduction:

I. Energy bands in semiconductors.

The energy levels of electrons in semiconductors are so close together that they can be considered as continuous energy bands. Semiconductors exhibit two bands, called a valence band and a conduction band (Figure 1). The valence band corresponds to the electrons that facilitate the chemical bonds between the atoms. These electrons are connected to the atoms and cannot flow as current. The conduction band electrons are free. These electrons and the vacancies (holes) in the valence band act as the charge carriers in semiconductors.

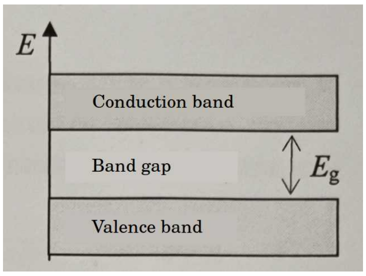

Figure 1

The valence and conduction bands are separated by an energy range where no electron states can exist. This range is called a band gap. The band gap E_g is different for every semiconductor. It can be interpreted as the minimum energy necessary to promote an electron from the valence band to the conduction band, i.e. to free it. This energy may be imparted in different ways:

- As thermal energy when the semiconductor is heated up. This is the reason why semiconductor conductivity increases with temperature. This property is used in thermistors.
- Through irradiation with light of the appropriate wavelength. This is used in photoresistors. The change in the conductivity $G \equiv \frac{1}{R}$ of the photoresistor due to the irradiation is proportional to the absorbed power,

$$\Delta G = C P_{\text{absorbed}}, \tag{1}$$

  where C is a constant that depends on the parameters of the photoresistor.

The aim of this problem is to determine the band gap of germanium (Ge) by studying a germanium photoresistor.

II. Spectral irradiance.

The spectral irradiance due to thermal radiation is defined as $f(\nu, t) = \frac{\mathrm{d}P}{S\mathrm{d}\nu}$, where $\mathrm{d}P$ is the power incident on an area S within a frequency range $\mathrm{d}\nu$ due to a source which subtends a solid angle Ω as viewed from S. According to Planck's law,

$$f(\nu, T) = \frac{2\Omega h\nu^3}{c^2\left(e^{h\nu/kT} - 1\right)}, \tag{2}$$

where h is the Planck constant, k is the Boltzmann constant, and c is the speed of light. At temperatures of up to 1500 K, in the near infrared and the visible spectrum we can assume $h\nu \gg kT$. Planck's law then simplifies to

$$f(\nu, T) \approx \frac{2\Omega h\nu^3}{c^2} e^{-h\nu/kT}. \tag{3}$$

The total power on S due to radiation of frequencies above some lower limit ν_0 can then be found by integrating Equation (3):

$$P(\nu > \nu_0) = S \int_{\nu_0}^{\infty} f(\nu, T)\,\mathrm{d}\nu = S \frac{2\Omega k T \nu_0^3}{c^2} e^{-h\nu_0/kT}. \tag{4}$$

Equipment:

1. Tungsten filament lamp of nominal voltage 12 V and nominal power 35 W. The lamp is fixed to a stand. There are two identically coloured wires at each terminal of the lamp. The wires will need to be connected to the circuit and the multimeters.
2. A germanium photoresistor fixed to the bottom of a cardboard box. The photoresistor is on the axis of the circular opening on the other side of the box. There are wires at the terminals of the photoresistor (Figure 2).
3. Three multimeters.
4. Rectifier that can supply constant voltage and constant current.
5. Two sheets of graph paper (you will not be given extra sheets).
6. Ruler.
7. Blank paper and tables (you can ask for extra sheets).

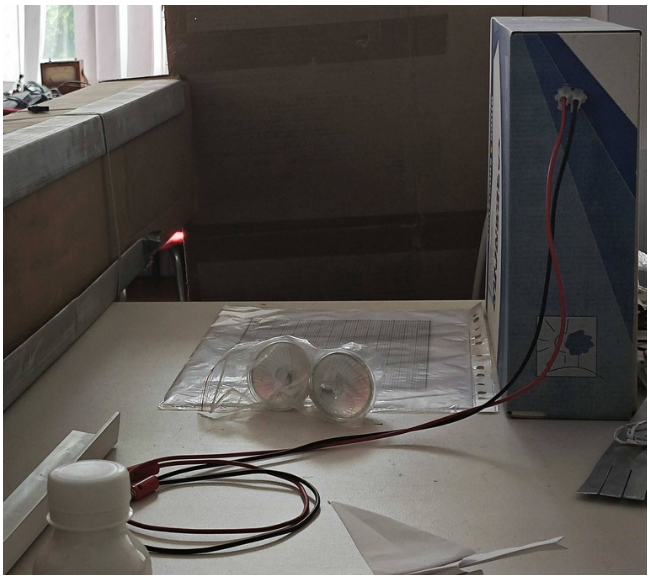

Figure 2

Tasks:

(a) Place the lamp against the circular opening of the box so that it can illuminate the photoresistor. Measure the resistance R_{bg} of the photoresistor when the lamp is turned off and there is only diffuse sunlight passing through the opening. Write down the value of R_{bg}.

(b) Assemble a circuit that can be used to find the resistance of the lamp at room temperature R_0. Sketch the circuit. Present your data in tabular and graphical form. Write down the value of R_0 along with its error.

(c) Study the dependence of the photoresistor's resistance R_{ph} on the temperature of the lamp's filament T. Use currents in the range (1 A, 3 A). Use $T_0 = 300\,\mathrm{K}$ for the room temperature. The resistance of tungsten changes with temperature as

$$R^{0.83} \propto T.$$

Present your results in tabular form. State the formulae you have used.

(d) Use the data from (c) to find the band gap of germanium E_{g}. Give your result in eV. Describe how you have analysed your data and submit the relevant tables and graphs.

(e) For what wavelengths can this photoresistor be used?

Constants:

Boltzmann constant	k	1.38×10^{-23} J/K
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Planck constant	h	6.63×10^{-34} J s

This problem is worth 15 points.
Time: 2.5 hours.

Solution.
(a) We connect the photoresistor with a multimeter in ohmmeter mode and obtain

$$\boxed{R_{\mathrm{bg}} = (61 \pm 1)\,\mathrm{k\Omega}.}$$

(b) It is clear that we need to measure the resistance at room temperature using multiple measurements and a graph. We do not want the lamp to heat up, so we need to minimise the power dissipation U^2/R. We assemble the circuit on Figure 3. We will use low voltages in the range (0 V, 0.35 V). The readings of the multimeters are given in the table below.

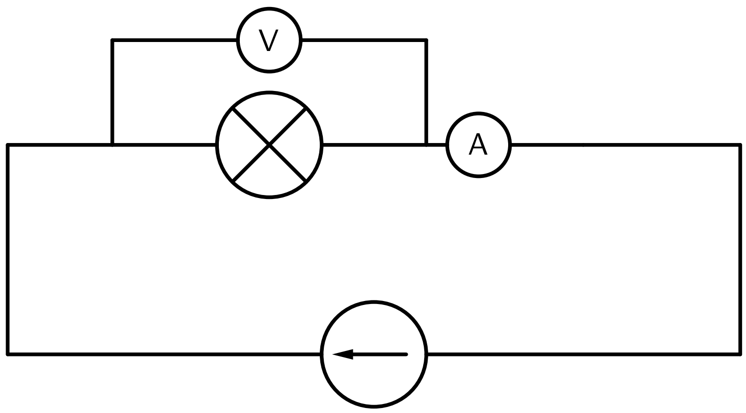

Figure 3

U, V	I, A
0.06	0.06
0.09	0.09
0.11	0.11
0.13	0.13
0.14	0.14
0.25	0.25
0.33	0.33

Table 1

Let us plot the voltage across the lamp U against the current I through it. The expected dependence is $U = IR_0$, so we can extract R_0 as the slope of the best fit line. From Figure 4 we find

$$R_0 = \frac{\Delta U}{\Delta V} = \frac{0.25\,\mathrm{V}}{0.25\,\mathrm{A}} = 1.00\,\Omega.$$

The error in R_0 will obviously be quite small. We can assume that it comes mainly from taking the slope inaccurately. Assume an error of 0.01 V in ΔU and 0.01 A in ΔI. Then, adding errors in quadrature,

$$\frac{\Delta R_0}{R_0} = \sqrt{\left(\frac{0.01\,\mathrm{V}}{0.25\,\mathrm{V}}\right)^2 + \left(\frac{0.01\,\mathrm{A}}{0.25\,\mathrm{A}}\right)^2} = 0.056.$$

Thus, rounding to one significant digit, $\Delta R_0 = 0.06\,\Omega$. Our answer is $\boxed{R_0 = (1.00 \pm 0.06)\,\Omega.}$

(c) We will use $T \propto R^{0.83}$ in the form

$$T = \frac{T_0}{R_0^{0.83}} R^{0.83} = \frac{T_0}{R_0^{0.83}} \left(\frac{U}{I}\right)^{0.83},$$

where U and I are the readings of the multimeters. We take measurements for the range (1 A, 3 A), with the results shown on Table 2.

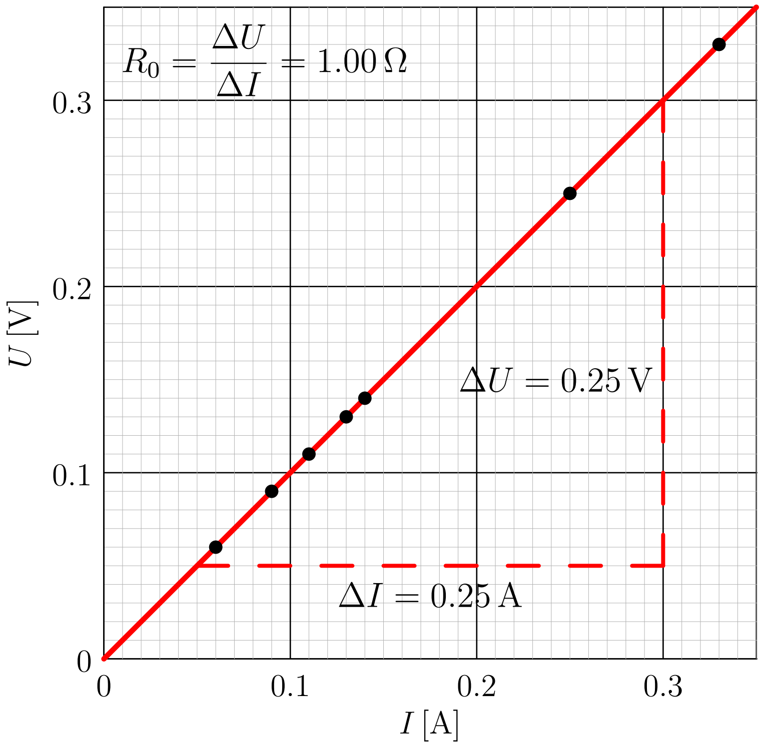

Figure 4

The main experimental consideration is that the photoresistor's conductivity will change not only due to the irradiation, but also due to heating. We seek to minimise the latter, which means that we should wait for a while between the measurements so that the system is kept at thermal equilibrium with the surroundings. Another less accurate option is to take the measurements very quickly so that the system does not heat up too much. This is what we have opted for here.

U, V	I, A	R_{ph}, kΩ	(U/I), Ω	T, K	x, $10^{-4} \times \mathrm{K}^{-1}$	y
3.49	1.42	2.28	2.46	633	15.80	1.68
3.73	1.50	1.54	2.49	639	15.65	2.29
4.33	1.60	0.97	2.71	687	14.56	2.69
4.82	1.70	0.67	2.84	714	14.01	3.03
5.82	1.90	0.37	3.06	759	13.18	3.57
6.45	2.00	0.30	3.23	795	12.58	3.73
7.56	2.20	0.21	3.44	837	11.95	4.04
8.32	2.32	0.17	3.59	867	11.53	4.21
9.54	2.50	0.14	3.82	912	10.96	4.36
10.90	2.70	0.11	4.04	957	10.45	4.54

Table 2

(d) First, we should recognise that the power P_{absorbed} in Equation (1) which causes in a change in conductivity is only due to the photons that can promote electrons to the conduction band. In other words, this is the power from photons of energies larger than E_{g}, i.e. of frequencies larger than ν_g, where $E = h\nu_g$. It follows that the change in conductivity at temperature T is given by

$$\Delta G = \frac{1}{R} - \frac{1}{R_0} = CS\frac{2\Omega k}{c^2}\nu_g^3 T e^{-h\nu_g/kT} \equiv ATe^{-h\nu_g/kT},$$

where A is some constant. This can be expressed as

$$\ln\left(\frac{1}{T}\left(\frac{1}{R} - \frac{1}{R_0}\right)\right) = A' - \frac{h\nu_g}{k}\left(\frac{1}{T}\right),$$

where $A' = \ln A$. This relation can now be linearised. Choose auxiliary variables

$$x \equiv \frac{1}{T}, \qquad y \equiv \ln\left(\frac{1}{T}\left(\frac{1}{R} - \frac{1}{R_0}\right) \cdot (\Omega\,\mathrm{K})\right).$$

The dependence $y(x)$ is then expected to be linear, with its slope corresponding to $-E_g/k$. The plot of $y(x)$ is shown on Figure 5. The slope is again found from the best fit line. We can estimate the error for the slope by drawing worst fit lines around it (one rule of thumb is that these should split the data points in a ratio of 2:1). The error for the slope is assumed to be half the difference between the slopes of the worst fit lines.

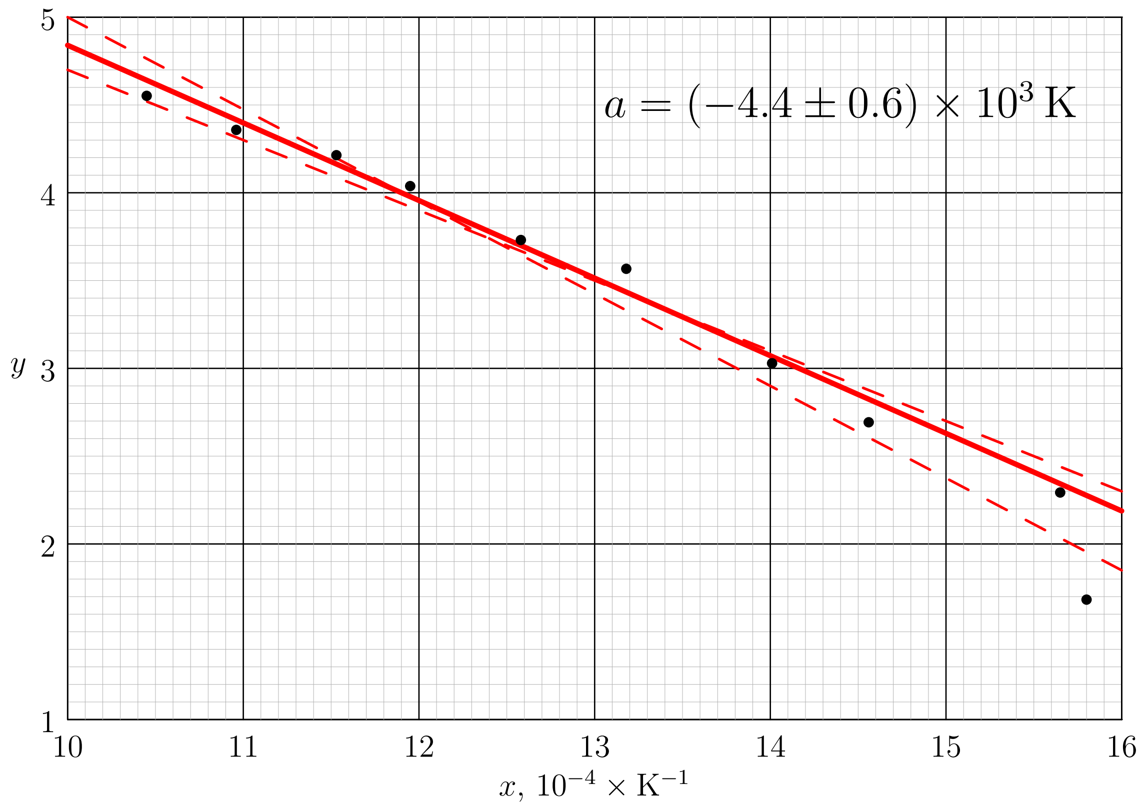

Figure 5

All that remains is to multiply our result for the slope by the Boltzmann constant. The band gap is found to be

$$\boxed{E_g = (0.38 \pm 0.05)\,\mathrm{eV}.}$$

The literature value for the band gap is around 0.7 eV. Considering our experimental setup and our methods, this is not a cause for concern. Indeed, all students that were within an order of magnitude received full marks.

(e) Again, any incident monochromatic light should have photons of energy above E_g. This corresponds to wavelengths

$$\lambda < \lambda_\mathrm{lim} = \frac{hc}{E_\mathrm{g}} = \boxed{(3.2 \pm 0.4)\,\mu\mathrm{m}.}$$

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