2019 Bulgarian IPhO Team Selection Test

Short Exam 1

Problem. One-dimensional crystal. A large number of balls (atoms) of mass m are positioned on a line at a distance a from each other. Adjacent balls are connected by identical springs of constant k. The balls are indexed with numbers $n = 0, 1, 2, \ldots$, where $n = 0$ corresponds to the ball at the left end of the chain (see Figure 1). The leftmost ball oscillates longitudinally with an amplitude A ($A \ll a$), as given by

$$x_0 = A \sin(\omega t),$$

where x_0 is the displacement from its equilibrium position. As a result, a longitudinal wave of wavelength λ propagates to the right. In what follows, you may use the magnitude of the wavevector $q = \frac{2\pi}{\lambda}$ instead of the wavelength λ.

Figure 1

(a) Find an expression for the displacement x_n of the n-th ball ($n > 0$) as a function of time.

(b) By considering the motion of the n-th ball ($n > 0$), find the dependence $\omega(\lambda)$ (or $\omega(q)$) of the wave's angular frequency on the wavelength λ (or the wavevector q).

(c) Acoustic waves have a wavelength much larger than the atomic distance, i.e. $\lambda \gg a$. The speed of sound in a crystal is essentially independent of the sound's frequency. Find an expression for the speed of the acoustic waves in the chain in terms of k, m, and a.

(d) The power of an acoustic wave is defined as the mean energy carried by the wave per unit time. Find an expression for the power of an acoustic wave P in terms of its angular frequency ω, its amplitude A, as well as the parameters k, m, and a.

Hint: You may use that

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right).$$

The problem is worth 5 points.
Time: 60 minutes.

Short Exam 2

Problem. Conducting sphere. The centre of a neutral conducting sphere is collinear with two point charges q and $-q$. The charges and the sphere are in vacuum. The distance between the sphere and the charges is l, the radius of the sphere is r, and the distance between the charges is d, such that $l \gg r$ and $l \gg d$. Find a formula for the force F on the sphere due to the charges.

The problem is worth 5 points.
Time: 60 minutes.

Short Exam 3

Problem. Surface gravity waves. A monochromatic surface gravity wave propagates along a channel of depth H and width $l \gg H$. The wavelength of the wave greatly exceeds the width of the channel. For such waves the relation between the angular frequency of the wave ω and the wavenumber $k = \frac{2\pi}{\lambda}$ is $\omega = uk$, where u is the wave velocity. Here u is independent of k.

(a) Propose a form for the dependence of the wavenumber k on depth H for such waves.

(b) Consider the reflection and the transmission of such a wave at a point where the channel suddenly changes depth from H_1 to $H_2 = 4H_1$. Compare the amplitudes of the reflected wave B and the transmitted wave C with that of the incident wave A.

(c) Find the reflection coefficient $R = \left(\frac{B}{A}\right)^2$ and the transmission coefficient $T = \frac{k_2}{k_1}\left(\frac{C}{A}\right)^2$. Verify that $R + T = 1$.

The problem is worth 5 points.
Time: 60 minutes.

Theoretical Exam

Problem 1. Unwinding a string. A string of length l is wound around a cylinder of mass m and radius R, where $l \gg R$. The cylinder is initially at rest on a horizontal plane along which it can only roll without slipping. The free end of the string A is at the top of the cylinder. A constant horizonal force F is applied at the free end of the string, and the cylinder starts rolling (Figure 2).

(a) Find the acceleration of the centre of the cylinder a.

(b) Find the velocity of the centre of the cylinder v when the string is fully unwound.

Problem 2. Spiral motion. A point of mass m moves under a central force. It begins its motion at a distance r_0 from the center of force. For some initial velocity of magnitude v_0 the particle moves along a spiral trajectory where the velocity vector maintains a constant angle θ with the radius vector (Figure 3).

(a) Find the equation of the trajectory in polar coordinates, i.e. find the dependence $r(\varphi)$ of the distance to the centre r on the angle of rotation of the radius vector φ. The equation may include r_0 and θ.

(b) Find an expression (up to a constant) for the potential energy W of the particle in terms of the distance to the centre r.

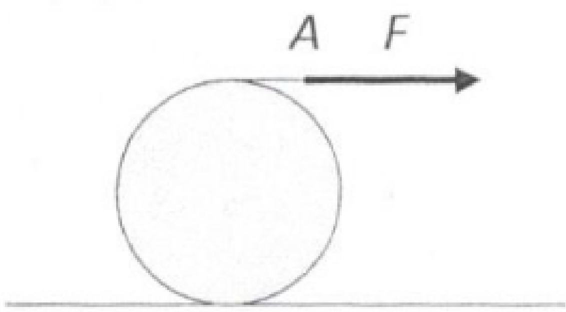

Figure 2

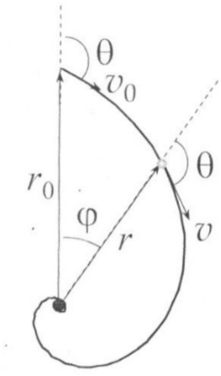

Figure 3

Problem 3. Hodograph. Let us draw the velocity vector $\mathbf{V}$ of a point mass on a diagram with axes V_x and V_y which correspond to the components of the velocity. If the velocity of the point mass varies, the end of the vector $\mathbf{V}$ will trace a curve called a hodograph (Figure 4). The hodograph can be interpreted as the trajectory of the vector $\mathbf{V}$ in velocity space.

(a) Draw the hodograph for a body launched at an angle α to the horizon with an initial velocity V_0. Mark the points which correspond to the launch and to the landing. Mark the point that corresponds to maximum height.

(b) Consider a simple string pendulum of length l with no friction at the pivot. The pendulum is initially at an angle of 90° to the vertical (Figure 5). The pendulum is then let go from rest. Draw the hodograph for the velocity of the bob *qualitatively.* Mark the points which correspond to maximum angular displacement. Mark the points which correspond to the pendulum's equilibrium position. Calculate the coordinates of the extrema of the hodograph. Annotate your diagram with your results.

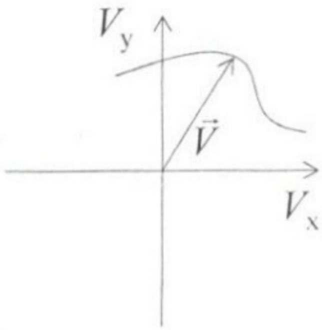

Figure 4

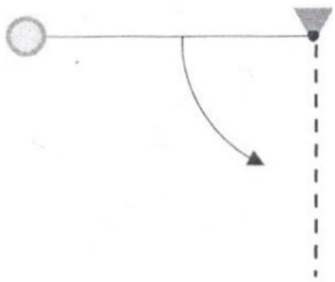

Figure 5

Problem 4. The Wien constant. For a black body of temperature T, the spectral radiance in frequency $r(\nu, T)$ (the energy emitted in a unit frequency interval per unit emitter area per target solid angle per unit time, $\frac{\mathrm{W}}{\mathrm{Hz \cdot m^2 \cdot sr}}$) is given by

$$r(\nu, T) = \frac{2h}{c^2} \frac{\nu^3}{e^{h\nu/kT} - 1}.$$

(a) Derive a formula for the spectral radiance in wavelength $r'(\lambda, T)$ (the energy emitted in a unit wavelength interval per unit emitter area per target solid angle per unit time, $\frac{\mathrm{W}}{\mathrm{m^3 \cdot sr}}$).

(b) Express the Wien constant b (from Wien's displacement law $\lambda_{\max} T = b$) in terms of the Planck constant h, the Boltzmann constant k, the speed of light c, and some unknown number A.

(c) Calculate the number A, rounded to five significant figures. Calculate the Wien constant, rounded to five significant figures.

Problem 5. Prism. The cross section of a prism with refractive index n is an isosceles trapezium. The angle between its larger base and its legs is θ, its height is h, and the length of the larger base is s. Consider a ray of light (lying in the plane of the cross-section) which is incident on one of the legs of the prism, at an angle α with respect to its normal. The ray is reflected by one of the bases and leaves the prism at the other leg. The prism is surrounded by air of refractive index unity.

(a) Derive a formula for the deviation angle φ of the ray due to the prism (that is, the angle between the incident ray and the outgoing ray).

(b) Assume that the incident ray is parallel to the bases. The ray strikes the prism at a distance x above the larger base. There exist ratios s/h for which the outgoing ray is also parallel to the bases for all $x \in (0, h)$.Find a formula for the smallest such ratio.

(c) Calculate the ratio s/h (as a decimal) for $\theta = 60.0^\circ$ and $n = 1.500$.

(d) Consider a prism with this ratio. You are looking at the Cyrillic letter Б through the legs. Draw what you will see.

Problem 6. Magnet. Assume the simplest possible model for the Earth's magnetic field: the source is located at the centre of the Earth, and its size is negligible compared to the radius of the Earth.

(a) Find the ratio of the magnetic fields at the magnetic poles and the magnetic equator.

(b) Find the angle which the magnetic field in Sofia (latitude 42.7°) makes with the horizon.

Problem 7. Photocathode. Compare the maximum velocities $v_{1,\max}$ and $v_{2,\max}$ of the photoelectrons for silver (work function $A = 4.7\,\mathrm{eV}$) in the following cases:

1. The photocathode is irradiated with ultraviolet light of wavelength $\lambda_1 = 155\,\mathrm{nm}$.
2. The photocathode is irradiated with gamma rays of wavelength $\lambda_2 = 2.47 \times 10^{-3}\,\mathrm{nm}$.

Problem 8. Hydrogen. A hydrogen atom at rest emits a photon in a transition from the first excited state to the ground state. What is the difference (in percent) between the energy of the emitted photon and the energy of the transition?

Problem 9. Accelerator. A particle accelerator works with particles of rest mass m_0. For what kinetic energies should the accelerator be designed if it is to be used for probing structures of size l? Make numerical estimates for electrons and protons, with $l = 10^{-15}\,\mathrm{m}$ (the length scale of atomic nuclei).

Problem 10. Helium.

(a) A vessel is filled with helium at temperature $T = 300\,\mathrm{K}$. A hole of size $S = 1\,\mathrm{mm}^2$ is cut in the vessel. What should the pressure in the vessel be so that the gas flows out from the hole at a rate of $\omega = 1\,\mathrm{g/h}$? Assume that the vessel is in vacuum and that the pressure inside does not change significantly.

(b) A disc of radius $r = 1\,\mathrm{cm}$ moves along its axis in a medium filled with helium at temperature $T = 300\,\mathrm{K}$ and pressure $p = 1\,\mathrm{kPa}$. Find the drag force acting on the disc.

Constants:

Acceleration due to gravity	g	$9.81\,\mathrm{m/s^2}$
Gas constant	R	$8.3\,\mathrm{J/mol\,K}$
Boltzmann constant	k	$1.380\,65 \times 10^{-23}\,\mathrm{J/K}$
Vacuum permittivity	ε_0	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Speed of light in vacuum	c	$2.997\,93 \times 10^{8}\,\mathrm{m/s}$
Elementary charge	e	$1.6 \times 10^{-19}\,\mathrm{C}$
Electron mass	m_e	$9.11 \times 10^{-31}\,\mathrm{kg}$
Proton mass	m_p	$1836 m_e$
Planck constant	h	$6.626\,07 \times 10^{-34}\,\mathrm{J\,s}$

Each problem is worth 3 points.
Time: 5 hours.

Experimental Exam

Problem 1. Separation of magnets.

Equipment:

1. Two identical neodymium magnets of size 15 mm × 10 mm × 5 mm and mass 5.6 g. They are magnetised parallel to their shortest edge.
2. Two identical plastic rails with an L-shaped cross section. The length of the rails is $L = 1.000\,\mathrm{m}$ and their thickness is $d = 3.0\,\mathrm{mm}$.
3. Spring scale with a range of 5 N and an adjustable zero. The scale can measure both tension and compression forces.
4. Plastic tape measure, accurate to 1 mm.
5. Stopwatch.
6. Ball of plasticine.
7. Two sheets of graph paper (you will not be given extra sheets).
8. Blank paper (you can ask for extra sheets).

Figure 6 shows two magnets in the shape of rectangular cuboids with edges a, b, and c. They are magnetised in the direction of edge c, i.e. their poles are on the parallel faces ab. If the magnets are placed with their unlike poles together, for small distances d between the poles the attractive force between the magnets is given by

$$F(d) = F_0 e^{-kd}, \tag{1}$$

where k is a constant which depends on the size of the magnets and F_0 is the so called breakaway force. This is the minimum force necessary to separate the magnets when their poles touch. The principal aim of this problem is to find the breakaway force for two neodymium magnets without a tool that can measure this force directly.

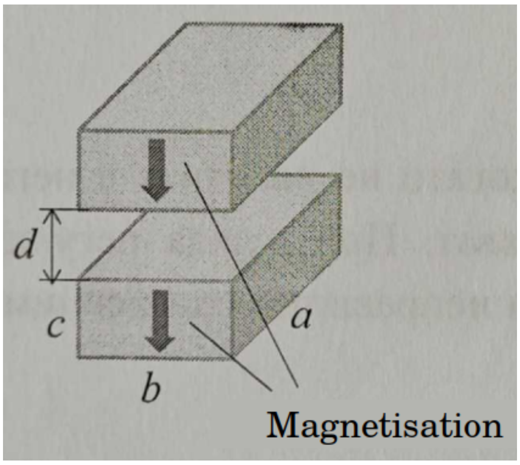

Figure 6

Tasks:

Design an experiment and determine:

(a) The coefficient of friction μ between the magnets and the plastic rail.

(b) The breakaway force F_0 between the two magnets.

Depending on your methods, you may not need some of the equipment. You can find the two parameters in any order. Your mark will depend on the explanation of your methods, the tabular and the graphical presentation of your data, your final values, and their error estimates.

Note: Do not load the spring scale when not working with it. Do not apply forces beyond the range of the spring scale. You may need to adjust the zero of the spring scale. If you damage the spring scale through your fault, you will not be given a spare.

Problem 2. AC Circuits.

Equipment:
Unknown resistance R, unknown capacitance C, unknown inductance L (these are inside a box, each between neighbouring terminals, respectively 1-2, 2-3, or 3-4, see Figure 7), multimeter (with instructions), alternating voltage source (with instructions), wires, ruler, graph paper.

Record all measurements in tables. Write down your results in the answer sheet.

Figure 7

Task 1. Determining the positions of the components.
Using only the multimeter, find out the positions of the resistor, the capacitor, and the inductor. Record your answers on the answer sheet. **(1.0 pt)**

Task 2. RC circuit measurements.
Set the alternating voltage source (the generator) to sine wave mode. Set the amplitude to maximum using the 'AMPL' potentiometer. Use the 'OUTPUT' terminal. The display will show the frequency $\nu = \frac{1}{T}$ of the output voltage. Use the multimeter's needle probles for faster measurements.

Notes:

1. The multimeter's output voltage is not constant. Its amplitude may vary depending on the load.
2. The multimeter measures alternating voltages accurately only in the range (40 Hz – 400 Hz). However, you can also use it for measuring much higher frequencies. Assume that the measured voltage U_{meas} is related to the true value U by $U_{\text{meas}} = k(\nu)U$, where $k(\nu)$ is some slowly decreasing function of the frequency ν.
3. In addition to the alternating voltage, there may be a constant voltage at the 'OUTPUT' terminal. Adjust the 'DC OFFSET' knob so as to minimise it. You can measure the output with the multimeter in DC voltage mode.

(a) Measure the resistance R. **(0.5 pt)**

(b) Assemble a circuit with R and C in series. Measure the dependence of the voltages U_R (across the resistor) and U_Z (across the RC circuit) on frequency. Work in the range (80 Hz – 800 Hz). Always measure U_R and U_Z in pairs at the same frequencies. Record your results in a table. **(2.0 pt)**

(c) Using appropriate variables, plot a linearised graph from which C can be calculated. The impedance of the RC circuit is $Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$, where $\omega = 2\pi\nu$ is the angular frequency. **(2.0 pt)**

(d) Using the graph, find the capacitance C. **(1.0 pt)**

Task 3. RLC circuit measurements.

(a) Assemble a circuit where C and L are in parallel, and this pair is in series with the resistance R. The impedance of such an RLC circuit is $Z = \sqrt{R^2 + \left(\frac{\omega L}{1-\omega^2 LC}\right)^2}$, with $\omega = 2\pi\nu$. This circuit exhibits resonance properties. Study the range (2 kHz – 20 kHz) and determine the resonant frequency ν_{res} with an accuracy of 100 Hz. **(1.5 pt)**

(b) Using the data obtained so far, find the value of L. **(1.0 pt)**

(c) Measure the dependence of the voltages U_R (across the resistor) and U_Z (across the RLC circuit) on frequency. Work in an appropriate range around ν_{res}. Always measure U_R and U_Z in pairs at the same frequencies. Record your results in a table. **(2.0 pt)**

(d) Using appropriate variables, plot a linearised graph. **(2.0 pt)**

(e) Using the graph, recalculate the resonant frequency ν_{res}, the capacitance C, and the inductance L. **(2.0 pt)**

Call the examiner in case of any technical difficulties.

Each problem is worth 15 points.
Time: 5 hours.