2017 Bulgarian IPhO Team Selection Test

Short Exam 2

Problem. A solid dielectric[1] sphere of radius R and relative permittivity $\varepsilon_r = 1$ is charged uniformly, such that the total charge is Q. The sphere is surrounded by a dielectric medium, also of permittivity $\varepsilon_r = 1$. The charge density in the medium varies with distance as $\rho(r) = \alpha/r$, where α is a known constant (and naturally, $r > R$).

(a) Find the charge Q which would make the electric field have a constant magnitude across the medium.

(b) Find the magnitude of this electric field E.

(c) A point charge q of mass m is located at a distance L from the centre of the sphere (q and Q are like). The charge is launched with a velocity v at an angle $\beta \in (0, \frac{\pi}{2})$ from the direction towards the centre of the sphere. For what velocities v will the charge avoid hitting the sphere? Neglect any frictional forces.

[1] In the original problem statement, the sphere was conducting. In that case, Part (c) becomes a complicated image charge problem which reduces to a fifth order equation with no analytical solution. Surely this isn't what the problem author intended.

Theoretical Exam

Problem 4. A resistance $R = 2\,\Omega$ and a nonlinear lightbulb are connected in parallel. They are connected to a battery of EMF $E = 4\,\mathrm{V}$ and internal resistance $r = 0.5\,\Omega$. The I-V curve of the lightbulb is given in the table. Find the power dissipated at the lightbulb.

U, V	I, A
0.50	0.60
1.00	1.00
1.50	1.30
2.00	1.55
2.50	1.75
3.00	1.90

Problem 5. The distance between a light source and a screen is L. There are two positions where a lens can be put between the source and the screen so that a clear image of the source is projected on the screen. The ratio of these images' sizes is k ($k > 1$). Find a formula for the focal length of the lens F in terms of L and k. Calculate F for $L = 180\,\mathrm{cm}$ and $k = 4$.

Problem 6. Two parallel beams of light are normally incident on a diffraction grating of period $d = 8\,\mu\mathrm{m}$. Their wavelengths are λ_1 and λ_2, both in the range $(400\,\mathrm{nm} - 760\,\mathrm{nm})$. We observe their diffraction patterns for all angles $\theta \in (-90°, 90°)$. The patterns' maxima coincide at an angle θ_0 with $\sin\theta_0 = 0.2725$. One of the patterns has 8 maxima more than the other. Find λ_1 and λ_2. How many possible solutions are there?

Experimental Exam

Problem 1. Deceiving the compass.

Equipment:

1. Compass with degree markings (Figure 1).
2. Small neodymium magnet in the shape of a rectangular cuboid. The edges of the magnet are, in ascending order,

$$a = 5\,\text{mm};\ b = 10\,\text{mm};\ c = 15\,\text{mm}.$$

   The mass of the magnet is 5.6 g, distributed uniformly. Its magnetisation is parallel to one of the edges, meaning that the poles are on the surfaces perpendicular to that edge.
3. Plastic ruler.
4. Stopwatch.
5. String.
6. Graph paper.
7. Marker pen.

Figure 1: Fingers.

The aim of this problem is to determine the horizontal component B_0 of the Earth's magnetic field, as well as the dipole moment p of the neodymium magnet.

Tasks:

(a) Design and describe an experiment that can be used to find whether the magnets are magnetised along a, b, or c. Also find the surfaces corresponding to the north pole (N) and the south pole (S) of the magnet. Use the marker to indicate them on the magnet using the letters N and S.

(b) Study the deviation of the compass from the north-south axis at different distances r between the needle and the neodymium magnet. The distance is measured between the centres of the objects. Using the data, plot a graph in the appropriate variables and use it to find the ratio p/B_0. Estimate your error.

(c) Design and carry out an experiment from which you can determine the product pB_0. Estimate your error.

(d) Calculate the horizontal component of the Earth's magnetic field B_0 and the dipole moment p of the magnet. Estimate your errors.

Note: Do not falsify your data so as to obtain the textbook value of B_0. You will be in for a nasty surprise!

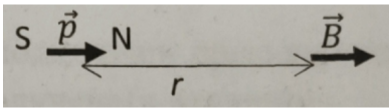
Relevant theory:
1) A magnetic dipole moment $\mathbf{p}$ can be ascribed to any current I circulating on a closed loop of surface S:

$$\mathbf{p} = IS\mathbf{n},$$

where $\mathbf{n}$ is a unit vector normal to the surface S. The units for dipole moment are $[\mathrm{A\,m^2}]$. The atoms in permanent magnets can be treated as tiny current loops, meaning that permanent magnents also possess a net dipole moment. It points from their south pole to their north pole.

2) The point dipole approximation holds when the distance to a dipole r is much larger than its characteristic length. Then the field along the axis of a dipole is given by

$$\mathbf{B} = \frac{\mu_0 \mathbf{p}}{2\pi r^3}.$$



3) When a dipole is placed in an external magnetic field $\mathbf{B}_{\mathrm{ext}}$, it experiences a torque

$$\mathbf{M} = \mathbf{p} \times \mathbf{B}_{\mathrm{ext}} \qquad \text{(i.e. of magnitude } M = pB_{\mathrm{ext}} \sin\theta).$$

that tends to rotate it towards the direction of the field.

4) The moments of inertia of a uniform rectangular cuboid about axes passing through its centre parallel to the edges a, b, and c, are respectively given by

$$I_a = \frac{1}{12}m(b^2 + c^2); \quad I_b = \frac{1}{12}m(c^2 + a^2); \quad I_c = \frac{1}{12}m(a^2 + b^2).$$

Problem 2. Conductivity of liquids.

Equipment:
Alternating voltage source, 2 multimeters, wires, paperclips of unknown radius r, tub, ruler, 3 bottles with 1.5 l of tap water in each, kitchen scale, table salt (NaCl), tape, scissors, aluminium foil (be careful with the sharp edges), funnel, graph paper.

All inductances and capacitances in the circuits below are negligible. Record all measurements in tables. Write down your results in the answer sheet.

Task 1. Conductivity of tap water.
Consider two identical infinite cylindrical conductors of radius r. The distance between the conductors is L. If the conductors pierce an infinite weakly conducting layer normally, some resistance R will be measured between them. If the thickness of the layer is h and its conductivity is σ, the resistance is given by

$$R = \frac{C}{\sigma h} \ln\left(\frac{L}{r}\right), \tag{1}$$

where C is an unknown constant.

Pour the three bottles of water into the tub. Assemble a circuit that will allow you to measure the resistance between two vertical paperclips in a layer of water. The multimeters should be connected in a way that allows you to measure small resistances. Use AC voltage of RMS value $U = 4\,\mathrm{V}$ and frequency $\nu = 2\,\mathrm{kHz}$.

1. Measure the resistance between the paperclips R at different distances $L \in [1\,\mathrm{cm}, 5\,\mathrm{cm}]$ in steps of $0.5\,\mathrm{cm}$. Keep the paperclips as far away as possible from the sides of the tub. Present your results in a table. **(4.0 pt)**

2. Stick a sheet of foil to each of the two smaller sides of the tub. Measure the resistance of the tap water between the sheets R. Find the conductivity of the tap water σ. Record the necessary length measurements in the answer sheet as well. **(2.0 pt)**

3. Using Equation (1) and your value for the conductivity of tap water σ from Task 1(b), plot your data from Task 1(a) in appropriate variables and find the radius of the paperclips r, as well as the constant C. **(3.0 pt)**

Task 2. Conductivity of salt salutions.
Add salt to the water and study four solutions of mass fractions $c = 0.50\,\%$, $1.00\,\%$, $1.50\,\%$, $2.00\,\%$ one after the other.

1. Use the sheets of foil from Task 1(b) to measure the conductivity σ of these solutions. Assume that it is given by

$$\sigma = \alpha c. \tag{2}$$

   Plot a graph of the conductivity σ against the mass fraction c of the salt solutions. Using the graph, find the parameter α. **(4.0 pt)**

2. Equation (2) does not agree well with the experimental data. Instead assume a more general dependence of the form

$$\sigma = \beta c^n. \tag{3}$$

   Plot another graph in the appropriate variables and use it to find the number n. **(2.0 pt)**

Call the examiner if you suspect that a multimeter's fuse has blown, or in case of any other technical difficulties.

Constants:

Vacuum permeability	μ_0	$4\pi \times 10^{-7}\,\mathrm{N/A^2}$