2015 Bulgarian IPhO Team Selection Test

Short Exam 2

Problem. Efficiency of a circuit. Consider the circuit on Figure 1. The voltage source, the ammeters, and the voltmeter are ideal. The resistances R_1 and R_2 are constant, while the resistance R_x may vary between zero and infinity. The efficiency of the circuit η is defined as the ratio of the power dissipated at R_x to the total power dissipated in the circuit.

(a) Find an expression for η in terms of R_1, R_2, and R_x. **(2.5 pt)**

(b) Find the value of R_x which maximizes η, in terms of R_1 and R_2. **(1.5 pt)**

(c) Find an expression for the maximum η in terms of the ratio $k = R_2/R_1$. **(0.5 pt)**

(d) Find the value of η (in percent) for $k = 1$. **(0.5 pt)**

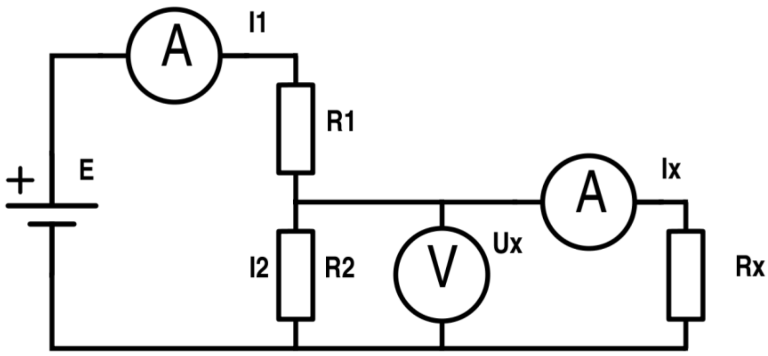

Figure 1

Problem 4. Alloy density. Gold-copper alloys Au_xCu_{1-x} (of number fraction x) have a face-centred cubic lattice, as shown on Figure 2. The atoms of copper and gold are randomly distributed on the lattice points. The density of pure gold is $\rho_{Au} = 19.30\,g/cm^3$, its lattice constant is $a_{Au} = 4.078\,Å$, while the density of pure copper is $\rho_{Cu} = 8.96\,g/cm^3$ and its lattice constant is $a_{Cu} = 3.615\,Å$. Assume that the lattice constant is proportional to x. Let y be the mass fraction of the alloys Au_xCu_{1-x}, such that $y = \frac{m_{Au}}{m_{Cu}+m_{Au}}$.

1. Find the number fraction in terms of the mass fraction, $x = f(y)$. **(1.0 pt)**
2. Find the density in terms of the mass fraction, $\rho_{Au_xCu_{1-x}} = f(y)$. **(1.0 pt)**
3. Find the density of a gold-copper alloy for mass fractions $y = 0.5$ and $y = 0.8$. **(1.0 pt)**

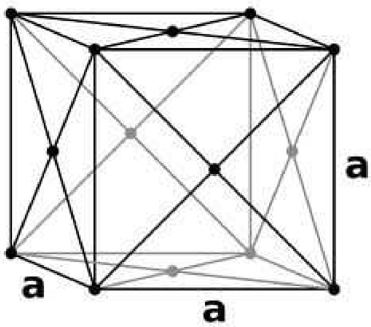

Figure 2

Problem 5. A microscopic model for resistance. The simplest microscopic model for resistance assumes that the free electrons in metals are accelerated from rest by the external electric field for time τ, after which they collide with the ionic lattice. This τ is called the mean free time. After colliding with the lattice, the electrons lose all of their velocity. Then they begin accelerating again.

(a) The electron number density in a metal is n and its resistivity is ρ. Express τ in terms of these parameters. **(1.0 pt)**

(b) Find the ratio $P/\frac{\Delta E_{\text{kin}}}{\Delta t}$ between the power P dissipated in a conductor of length l and cross section S, and the kinetic energy $\frac{\Delta E_{\text{kin}}}{\Delta t}$ lost by the electrons (as heat) per unit time. **(1.0 pt)**

(c) Find the mean free time for the electrons in aluminium. Aluminium ions have charge $q = +3e$. The atomic mass of aluminium is $A = 27.0$, its density is $\mu = 2.70\,g/cm^3$, and its resistivity is $\rho = 28.2 \times 10^{-9}\,\Omega\,m$. **(1.0 pt)**

Problem 6. Charged disc in a magnetic field. A uniform dielectric disc of mass m and charge q is initially at rest. The disc is placed in a magnetic field parallel to its axis. The field is arbitrary, $B = B(t)$, with $B(0) = 0$. Find the time dependence of the angular velocity of the disc $\omega(t)$.

Experimental Exam

Problem 1. Efficiency of a circuit.

Equipment:
DC voltage source (rectifier), 3 multimeters, resistor substitution box R_x, two resistors R_1 and R_2 in series, 8 wires, graph paper. See Figure 3.

Figure 3

Let us define the efficiency of the circuit on Figure 4 as the ratio of the power dissipated to the right of the voltmeter (i.e. at the resistor R_x and the ammeter I_x) to the total power dissipated in the circuit. The aim of this problem is to study the dependence of η on the load R_x (which will be calculated as $R_x = U_x/I_x$). This will be used to find the optimal value $R_{x,\text{opt}}$ which maximises η (accurate to $5\,\Omega$), as well as the maximum η itself.

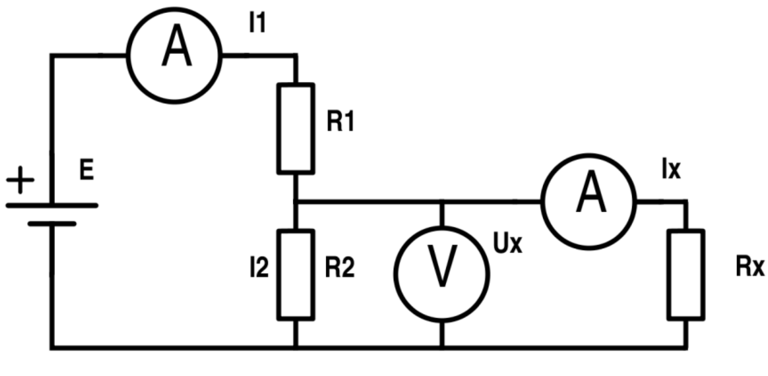

Figure 4

(a) Using a multimeter in ohmmeter mode, measure the resistances R_1 and R_2 and record them in a table. Turn on the voltage source in DC mode and set the voltage to $E = 2.00\,\text{V}$. Leave the source on and do not change the voltage from now on. **(1.0 pt)**

(b) Assemble the circuit. Before connecting it to the voltage source, ask the examiner to confirm. **(3.0 pt)**

(c) Assume that the dependence $\eta(R_x)$ has a single maximum. Choose appropriate values for R_x in the range $(0.1R_x - 10R_x)$. Take the necessary measurements and tabulate the dependence for this range. In the table you should record both the nominal resistance of the substitution box and its measured value $R_x = U_x/I_x$. **(5.0 pt)**

(d) Plot a graph of $\eta(R_x)$. **(2.0 pt)**

(e) If your experimental data is insufficient, take additional measurements. If the scale of your graph was unsuitable, plot an additional graph which covers only the important features of the dependence. Write down your values for $R_{x,\text{opt}}$ and $\eta(R_{x,\text{opt}})$. **(4.0 pt)**

Call the examiner in case of any technical difficulties.

Constants:

Elementary charge	e	1.6×10^{-19} C
Electron mass	m_e	9.11×10^{-31} kg
Atomic mass unit	u	1.66×10^{-27} kg