2013 Bulgarian IPhO Team Selection Test

Theoretical Exam

Problem 4. A layer of tap water is poured into a wide and shallow dielectric vessel. The thickness of the layer is $h = 1.00\,\mathrm{cm}$. The probes of a multimeter are vertically submerged at a distance $l = 20.0\,\mathrm{cm}$ from each other until they reach the bottom of the vessel. The multimeter measures a resistance of $R = 500\,\mathrm{k\Omega}$. Assuming the probes are cylinders of radius $a = 1.00\,\mathrm{mm}$ and modelling the water as a weakly conducting medium, find a formula for the resistivity of water ρ and calculate its value.

Problem 5. A square frame of side a and resistance R is placed next to an infinite conducting wire carrying a current I. The wire lies in the plane of the frame and is parallel to two of its sides. The distance from the wire to the near end of the frame is d ($d \gg a$). The frame starts to move away from the wire with constant velocity v (in the plane of the frame and perpendicular to the wire). Find a formula for the total heat Q dissipated at the frame until it is infinitely far from the wire. Calculate Q for $a = 2\,\mathrm{cm}$, $R = 0.01\,\Omega$, $d = 20\,\mathrm{cm}$, $I = 10\,\mathrm{A}$, $v = 5\,\mathrm{cm/s}$. The inductance of the frame is negligible.

Problem 6. A monochromator consists of:

- An isosceles glass prism of base $b = 10.0\,\mathrm{cm}$ and an angle $\theta = 50^\circ$ opposite the base.
- A thin entrance slit.
- An identical exit slit.
- A lens at the entrance which gives rise to a parallel beam incident on the prism.
- An identical lens which focuses the dispersed light onto the exit slit. The lenses are of diameter $D = 10.0\,\mathrm{cm}$ and focal length $f = 50.0\,\mathrm{cm}$.

The glass of the prism has a refractive index $n = 1.73$. Its dispersion at wavelength 550 nm is $\frac{\mathrm{d}n}{\mathrm{d}\lambda} = 1.35 \times 10^5\,\mathrm{m}^{-1}$. Inside the prism light travels parallel to the base. Using the Rayleigh criterion, find the theoretical upper limit for the spectral resolution of the monochromator $R = \lambda/\Delta\lambda$ (due to diffraction). Calculate R.

Experimental Exam

Problem 1. Ruler vibrations.

Equipment:
Ruler (50 cm), another ruler (shorter), clamp, wooden block, 3 old coins (Bulgarian, issue 1992, one is 5 leva, the other two are 1 lev), stopwatch, tape measure, Scotch tape, scissors, graph paper.

Data:

Mass of the ruler	m_L	51.0 g
Thickness of the ruler	h	3.00 mm
Mass of the 5 leva coin	m_5	6.00 g
Mass of a 1 lev coin	m_1	4.00 g

You will need to complete five tasks.

(a) Find the density of the plastic ρ that the ruler is made from. The ruler's cross section can be approximated as a parallelogram. **(1.5 pt)**

Clamp one end of the ruler so that the clamped part is horizontal and the free part is $L_0 = 47.0\,\text{cm}$ long. Load the ruler at its free end using different sets of coins, taking care that the centre of mass of the coins is exactly at the end of the ruler. Measure the additional deflection of the ruler s due to the coins. This deflection is given by

$$s = \frac{4FL_0^3}{Ebh},$$

where F is the force at the end of the ruler which causes the deflection, L_0 is the length of the ruler, b is its width, h is its length, and E is the Young modulus of the plastic. This formula applies for a rectangular cross section, but it is accurate enough in the case of our ruler as well.

(b) Plot a graph of the deflection s against the mass of the coins m. Using the graph, find the Young modulus of the plastic E. **(3.0 pt)**

Study the dependence of the oscillation period T of the free end of the ruler (without loading it with coins) on the free length L. This dependence is described by the formula

$$T = \frac{CL^2}{h}\sqrt{\frac{\rho}{E}},$$

where L is the free length, h is the thickness of the ruler, and C is some unknown constant.

(c) Plot a linearised graph in the appropriate variables. Using the graph, find C. Use the values of ρ and E from the previous parts of the problem. **(4.5 pt)**

Clamp the ruler at one end so that it is vertical, with the part that sticks out being $L_0 = 47.0\,\text{cm}$ long. Attach different sets of coins at its free end, taking care that the centre of mass of the coins is exactly at the end of the ruler.

(d) Study the dependence of the oscillation period T of the free end of the ruler on the total mass of the attached coins m. Plot your data in a way that will allow you to predict the period T_{20} for $m = 20\,\text{g}$ by extrapolating. **(4.0 pt)**

(e) Find the critical mass M at which the equilibrium of the vertical ruler becomes unstable and the ruler stops oscillating. **(2.0 pt)**

Call the examiner in case of any technical difficulties.

Problem 2. Induction brake.

Introduction:

A permanent magnet moving along a non-magnetic metal surface will induce eddy currents in the metal. These give rise to a magnetic field that results in a drag force $\mathbf{F}$ on the magnet:

$$\mathbf{F} = -k\mathbf{v},$$

where $\mathbf{v}$ is the velocity of the magnet and k is the magnetic drag coefficient. This coefficient depends on the shape and size of the magnet, the magnetic field of the magnet, and the conductivity of the metal surface. The aim of this problem is to find k for a magnet moving along an inclined aluminium surface.

Equipment:

1. Aluminium rail with U-shaped cross section (Figure 1).
2. Neodymium magnet of mass 5.6 g in the shape of a rectangular cuboid. It is magnetised parallel to its shortest edge, i.e. its poles are on its largest surfaces.
3. Stopwatch.
4. Plastic tape measure, accurate to 1 mm.
5. Pencil.
6. Ball of plasticine.
7. Ruler.
8. Blank paper and two sheets of graph paper.

Note: The magnet is made from a brittle alloy and will easily break if struck. Take care not to break it.

Tasks:

(a) Write down the equation of motion for a magnet on a metal surface at an angle α to the horizon. The magnetic drag coefficient is k and the coefficient of dynamic friction is μ. Find an expression for the terminal velocity of the magnet v_t assuming the rail is long enough.

(b) Place the rail an an angle of 60° to the horizontal table. Explain how you have found the angle. You can affix the lower end of the rail to the plasticine and lean the upper end on the wall. Place the magnet with one of its poles lying on the aluminium surface. Let the magnet slide with zero initial velocity from different initial heights. Study the dependence of distance moved s against time t. Present your data in tabular and graphical form.

(c) Analyse the graph from (b). State the type of motion within the time intervals you worked with. Find the terminal velocity of this motion v_t.

(d) Study the descent of the magnet at different slopes $\alpha < 60^\circ$ (though large enough for the magnet to slide). Find the terminal velocity v_t for each of the angles α. Present your results in a table.

(e) Find auxiliary variables x and y (expressed in terms of the slope α and the terminal velocity v_t) for which a linear dependence is expected. Present your data in terms of these variables in tabular and graphical form. Using your graph, find the coefficient of dynamic fricion μ and the magnetic drag coefficient k.

Figure 1

(f) The time dependence for the velocity of the magnet is given by

$$v(t) = v_{\mathrm{t}}(1 - e^{-t/\tau}), \tag{1}$$

where τ is a time constant which is independent of the slope (that is, the time taken to reach 63.2 % of the terminal velocity). Using your data, find the time constant τ. Is it possible to verify Equation (1) experimentally using only the equipment given?

Constants:

Acceleration due to gravity	g	$9.81\,\mathrm{m/s^2}$
Vacuum permeability	μ_0	$4\pi \times 10^{-7}\,\mathrm{N/A^2}$