2011 Bulgarian IPhO Team Selection Test

Short Exam 2

Problem. A circuit consists of a battery of EMF E, a capacitor C, a resistor R, and an open switch, all in series. At time $t = 0$ the switch is closed.

(a) Find the magnitude $I_1(0)$ of the initial current, the time dependence of the current $I_1(t)$, and the total heat Q_1 dissipated at the resistor until the capacitor is fully charged. **(2.0 pt)**

The capacitor is discharged and the resistor is replaced by a nonlinear element with an I-V curve $I = \beta U^{3/2}$, where β is a constant. At time $t = 0$ the switch is closed.

(b) Find the magnitude $I_2(0)$ of the initial current, the time dependence of the current $I_2(t)$, and the total heat Q_2 dissipated at the nonlinear element until the capacitor is fully charged. **(3.0 pt)**

Theoretical Exam

Problem 4. We wish to make a solenoid of inductance $L = 1\,\mathrm{mH}$ and length $l = 1\,\mathrm{m}$. The diameter of the solenoid is $d \ll l$.

(a) Find the length of the wire x that we will need. **(1.5 pt)**

(b) If the wire is made of copper and has a resistance of $R = 1.7\,\Omega$, find the mass of the solenoid m. **(1.5 pt)**

The density of copper is $\rho_m = 8.9\,\mathrm{g/cm^3}$ and its resistivity is $\rho_R = 17\,\mathrm{n\Omega\,m}$.

Problem 5. A particle of mass m and charge q is located at the origin of a Cartesian coordinate system at time $t = 0$. Around the particle there is a constant homogeneous magnetic field B along Oz and an oscillating homogeneous electric field $E(t) = E_0 \sin(\omega t)$ along Ox, with $\omega = \frac{qB}{m}$. Assume that the particle moves along an Archimedean spiral given by $r(\varphi) = A\varphi$ in polar coordinates. Here A is a constant and φ is measured starting from Ox. If the particle moves with a constant angular velocity ω, find the increase in the radius vector per turn.

Problem 6. A parallel beam of monochromatic light (wavelength $\lambda = 500\,\mathrm{nm}$) is normally incident on a diffraction grating. The period of the grating is $d \gg \lambda$. A convex lens of focal length $f = 1\,\mathrm{m}$ is placed parallel to the grating. A diffraction pattern is then observed on a screen in the focal plane of the lens. We now rotate the diffraction grating at an angle α with respect to an axis parallel to the slits. The new fourth order maximum is now observed where the fifth order maximum originally was.

(a) Find the angle α. **(1.0 pt)**

(b) Given that the fifth order maximum has shifted by $\Delta l = 12.5\,\mathrm{mm}$ along the screen due to the rotation, find the period of the grating d in micrometres. **(1.0 pt)**

(c) The screen is $L = 20\,\mathrm{cm}$ wide and positioned symmetrically with respect to the optical axis. Find the number of maxima N that will be observed on the screen after the rotation. **(1.0 pt)**

Experimental Exam

Problem 1. Oscillations of a wooden block.

Equipment:
Stand with pinch (to be used as a pivot for a pendulum), wire, wooden block, pliers, stopwatch, ruler, blank paper, graph paper.

The longest side of the block is denoted by a, the middle length one by b, and the shortest one by c. The mass of the block m is written down on the block itself. A hook with a screw thread can be inserted into holes on the surfaces of the block. This allows the block to oscillate about different axes. Denote the axes about which the block oscillates by [100] for the axis parallel to a, [010] for the axis parallel to b, and [001] for the axis parallel to c.

The aim of this problem is to calculate the acceleration due to gravity g **(10 pt)** and the torsion coefficient of the wire D **(5 pt)**. The two tasks are independent.

In order to find g you will need to study the period of rotational oscillations T of the pendulum about a horizontal axis which is perpendicular to one of the block's surfaces. The period is given by the formula

$$T = 2\pi\sqrt{\frac{I}{mgd\left(\frac{d}{l}+1\right)}},$$

where l is the length of the wire, d is the distance between the end of the wire and the centre of mass of the block, m is the mass of the block, and I is the moment of inertia of the block with respect to the axis of rotation. The moment of inertia is given by

$$I = \frac{1}{12}m(a_i^2 + a_j^2),$$

where a_i and a_j are the lengths of the edges perpendicular to the axis of rotation (i.e. a and b, or a and c, or b and c).

(a) Take enough useful measurements. Present them in a table and explain how they were obtained. **(4.5 pt)**

(b) State the variables which, when plotted, can easily give you g. **(0.5 pt)**

(c) Plot the relevant graph. **(3.0 pt)**

(d) Using the graph, determine the acceleration due to gravity g. **(1.0 pt)**

(e) Estimate your error in finding g. **(1.0 pt)**

In order to determine the torsion coefficient of the wire D you will need to study the period of rotational oscillations T of the pendulum about an axis coinciding with the wire. Their period is given by

$$T = 2\pi\sqrt{\frac{I}{D}},$$

where I is the moment of inertia about the axis of rotation (given above).

(f) Take enough useful measurements. Present them in a table and explain how they were obtained. **(2.25 pt)**

(g) State the variables which, when plotted, can easily give you D. **(0.25 pt)**

(h) Plot the relevant graph. **(1.75 pt)**

(i) Using the graph, determine the coefficient of torsion of the wire D. **(0.5 pt)**

(j) Estimate your error in finding D. **(0.25 pt)**

Call the examiner in case of any technical difficulties.

Note: Do not write on the block!

Constants:

Vacuum permeability μ_0 $4\pi \times 10^{-7}\,\mathrm{N/A^2}$