# 2006 Bulgarian IPhO Team Selection Test

## Short Exam 1

**Problem.** A uniform solid cylinder of mass $m$ and radius $r$ lies on its side on a horizontal surface. The cylinder is attached at its topmost point to a pair of identical horizontal springs, each with a spring constant $k$ and neglibible mass. The springs are perpendicular to the axis of the cylinder and are relaxed when the cylinder sits at its equilibrium position. There is zero friction when slipping. The acceleration due to gravity is $g$. Find the period of small oscillations of the cylinder.

*The problem is worth 5 points.*
*Time: 40 minutes.*

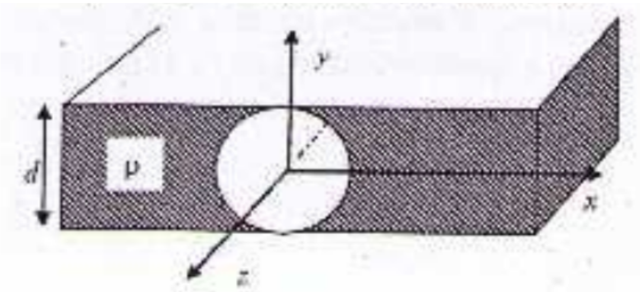
## Short Exam 2

**Problem.** A layer of thickness $d$ (infinite along the plane $zx$) carries a charge density $\rho$. A cylindrical cavity is dug out of the layer along the $z$-axis. Its diameter is equal to the thickness of the layer.

(a) Find expressions for the components $E_x$ and $E_y$ of the electric field inside the cavity.

(b) Find an expression for the potential $\varphi$ inside the cavity.

Express your answers through the coordinates $x$ and $y$ and the parameters above.




*The problem is worth 5 points.*
*Time: 40 minutes.*

## Short Exam 3

**Problem.** The curve of equilibrium for the liquid and the solid phase of helium-3 passes through a point of coordinates $p_1 = 31\,\mathrm{atm}$, $T_1 = 0.12\,\mathrm{K}$ on the $p$-$T$ diagram.

(a) Find the curve of equilibrium $p(T)$. At low temperatures the molar entropy of the liquid phase is given by $S_{\mathrm{liq}} = R\frac{T}{\Theta}$, with $\Theta = 0.46\,\mathrm{K}$, whereas the molar entropy of the solid phase does not depend on temperature and is given by $S_{\mathrm{sol}} = R\ln 2$. The difference between the molar volumes of liquid and solid helium-3 is constant and equal to

$$\Delta V = V_{\mathrm{liq}} - V_{\mathrm{sol}} = 1.25\,\mathrm{cm}^3/\mathrm{mol}.$$

(b) At temperature $T_2 = 0.42\,\mathrm{K}$, what is the pressure $p_2$ at which liquid and solid helium-3 are in equilibrium?

(c) At temperature $T = 0\,\mathrm{K}$, what pressures permit only a solid phase?

(d) Consider the line $p_1 = 31\,\mathrm{atm}$ within the range $0\,\mathrm{K} < T < 1\,\mathrm{K}$. Describe the possible phase transitions.

*The problem is worth 5 points.*
*Time: 40 minutes.*

## Theoretical Exam

**Problem 1.** Find the moment of inertia $I$ of a uniform regular triangle of side $l$ with respect to an axis passing through its centre of mass perpendicularly to its plane.
*Hint:* Do not use calculus.

**Problem 2.** Consider a biconvex symmetrical spherical thin lens. The reflections of distant objects from the lens give rise to two images which can be observed in the lens. When the eye of the observer is at a distance $l$ away from the lens, the two images seem identical in size. Find the radius of curvature $R$ and the optical power $\Phi$ of the lens. The refractive index of the glass is $n$. Calculate $R$ and $\Phi$ for $n = 1.5$ and $l = 12\,\text{cm}$.

**Problem 3.** A thin disc of radius $R$ can rotate around an axis passing through its centre perpendicularly to its plane. The disc is placed in a cylindrical cavity of radius slightly greater than $R$. The lateral surfaces of the disc and the cavity are parallel and the distance between them is $h$. The cavity is filled with a liquid of viscosity $\eta$. Find the power $P$ that an external force must supply to the axis of the disc so that it rotates with a constant angular velocity $\omega$. Ignore all edge effects. Calculate $P$ for $R = 10\,\text{cm}$, $h = 1\,\text{mm}$, $\eta = 8\times 10^{-3}\,\text{Pas}$, and $\omega = 10\,\text{rotations/s}$.

**Problem 4.**

(a) Find an expression for the magnetic field $B$ at point $O$ due to the conducting loop $ABCD$. Express your answer in terms of the parameters $I$, $a$, $\varphi$, $r$, and the magnetic constant $\mu_0$.

(b) For the case $a \ll r$, express your answer for $B$ in terms of $p$, $r$, and $\mu_0$, where $p = IS$ is the magnetic dipole moment of the loop, $S$ being the surface area of the loop.

(c) A long wire carrying current $I_1$ passes through $O$ perpendicularly to the loop. Find the torque $M$ acting on the wire and draw its vector on Figure 1. Answer in terms of $I$, $I_1$, $a$, $\varphi$, $r$, and $\mu_0$.

**Problem 5.** A metal ball of radius $a$ and charge $q_0$ is grounded through a resistance $R$, as shown on Figure 2.

(a) After closing the switch, find the time $T$ until the charge of the ball decreases by half.

(b) Find the heat $Q$ dissipated within this time interval.

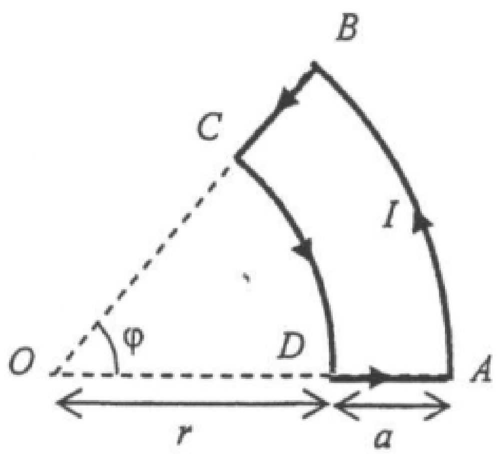



Figure 1

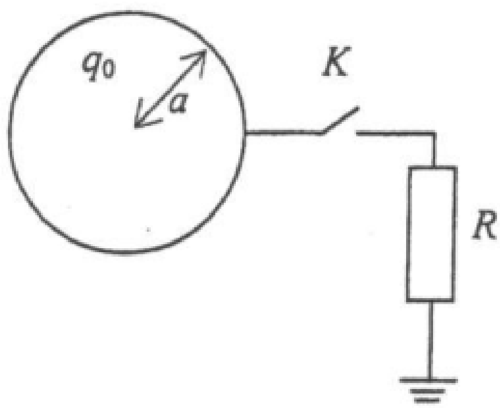



Figure 2

**Problem 6.** A neutral particle $X$ decays into two photons of energies $E_1 = 53\,\mathrm{MeV}$ and $E_2 = 172\,\mathrm{MeV}$ moving at right angles to each other.

(a) Find and calculate the velocity $v$ of the particle $X$ before the decay (as a fraction of $c$).

(b) Find and calculate the rest mass $M$ of the particle $X$ (in MeV).

**Problem 7.** Estimate the specific heat capacity of a copper coin at room temperature (for our purposes, room temperature means high temperature). The atomic mass of copper is 63.5 and its density is $\rho = 8.9\,\mathrm{g/cm^3}$.

**Problem 8.** At low temperatures (about $30\,\mathrm{K}$) the thermal properties of a dielectric crystal are governed by the atoms' harmonic oscillations. These oscillations are of a quantum nature, and correspond to waves that propagate through the crystal with velocity $5 \times 10^3\,\mathrm{m/s}$. The ensemble of oscillating atoms can be modelled as an ideal phonon gas. The phonons within the volume of the crystal give rise to a pressure $p$ on its faces given by $p = u(T)/3$, where $u(T)$ is the internal energy density. Provide an order-of-magnitude estimate for the pressure of the phonon gas.

**Problem 9.** Classical physics allows for an atomic nucleus to attract an electron close to itself. On the other hand, the Heisenberg uncertainty principle implies that the electron's kinetic energy would then increase sharply, making its rest energy negligible. Find the atomic number of the transuranium element which can keep an electron close to its nucleus. Assume that this element is stable.

**Problem 10.** Using the Bohr model for the hydrogen atom, calculate the magnetic field which a ground state electron creates at the centre of the atom.

**Constants:**

| | | |
|---|---|---|
| Acceleration due to gravity | $g$ | $9.81\,\mathrm{m/s^2}$ |
| Boltzmann constant | $k$ | $1.38 \times 10^{-23}\,\mathrm{J/K}$ |
| Vacuum permittivity | $\varepsilon_0$ | $8.85 \times 10^{-12}\,\mathrm{F/m}$ |
| Vacuum permeability | $\mu_0$ | $4\pi \times 10^{-7}\,\mathrm{N/A^2}$ |
| Elementary charge | $e$ | $1.6 \times 10^{-19}\,\mathrm{C}$ |
| Planck constant | $h$ | $6.6 \times 10^{-34}\,\mathrm{J\,s}$ |

*Each problem is worth 3 points.*
*Time: 5 hours.*

# Experimental Exam

**Problem 1. Measuring capacitance with an ohmmeter and a stopwatch.** The equivalent circuit of an ohmmeter is shown on Figure 3. The ohmmeter is made up of a volatage source $E$, an ammeter $A$, and an internal resistance $r$. When we measure an external resistance $R$, the value displayed by the ohmmeter is calculated from the current passing through $A$.

Should we connect a capacitor $C$ instead of a resistance $R$ (Figures 4 and 5), the voltage on the capacitor depends on time as

$$U(t) = E\left(1 - e^{-t/RC}\right).$$

(a) Obtain a formula for the reading of the ohmmeter $R(t)$ as a function of time ($t = 0$ when the circuit is closed).

(b) Using that $e^x \approx 1 + x + \frac{x^2}{2}$ when $x \ll 1$, obtain an approximate formula for $R(t)$.

(c) Find variables $x(R,t)$ and $y(R,t)$ such that the dependence of $y$ on $x$ is linear.

(d) Use the ohmmeter's $20\,\mathrm{k\Omega}$ range. At the start of your measurements connect the black terminal of the capacitor to the 'COM' terminal of the ohmmeter, and connect the other terminal of the capacitor to the '$\Omega$' terminal of the ohmmeter. Collect enough ohmmeter readings $R(t)$. Remember to discharge the capacitor at the start of a new series of measurements.

(e) Plot your data using the variables $x(R,t)$ and $y(R,t)$. Using a linear fit, calculate the capacitance $C$ and the internal resistance $r$ of the ohmmeter.

(f) Using your plot, provide error estimates for your values of $C$ and $r$.

**Note:** Do not peel the tape off the capacitor, or you will be disqualified.

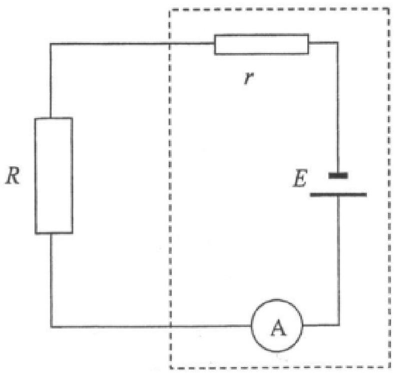



Figure 3

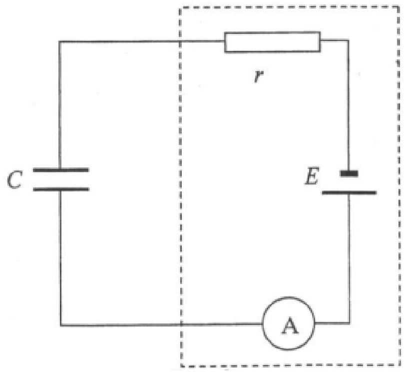



Figure 4

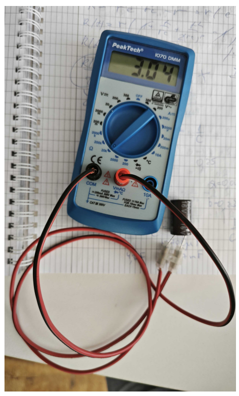


Figure 5

**Problem 2. Measurements with a dart.** You are given a dart of mass $m = 6.32\,\mathrm{g}$. It consists of a plastic body, at the tip of which there is a pillbox-shaped permanent magnet with a dipole moment along the axis of the dart (Figure 6). The centre of mass $C$ is located exactly where the metal part meets the plastic part. You are also given an inductor consisting of 4 coils of radius $r = 6\,\mathrm{cm}$ each. The inductor can be supplied with direct current from a rectifier. In addition, you have a ruler, a stopwatch, pins, strings, and a cardboard box, to be used as a stand (Figure 7).

Figure 6

(a) Using some string, make a torsional pendulum. Use it as a compass and find the north-south direction at your workstation. Keep any ferromagnetic objects away from the dart.

Now, adjust your stand so that the north-south line is perpendicular to the plane of the inductor. Set the pendulum so that the magnet of the dart is exactly at the centre of the inductor. Connect the coil to the rectifier.

(b) Study the dependence of the pendulum's period $T$ (of horizontal torsional oscillations) on the current $I$ through the inductor. Use currents within the range $[-4\,\mathrm{A}, 4\,\mathrm{A}]$.

(c) Plot the dependence of the period on the current in variables which would linearise it. Any torques due to the twist of the string can be neglected.

(d) Using your plot, find the Earth's magnetic field $B$ at your workstation.

Attach the pendulum to a vertical pin (having pierced the box with the pin). It will stick to the pin due to the attractive magnetic forces.

(e) Measure the oscillation period $T_O$. Calculate the moment of inertia $J_O$ of the dart about the pivot, i.e. the tip of the pin.

(f) Calculate the moment of inertia $J_C$ of the dart about an axis passing through its centre of mass and perpendicular to the dart's axis.

(g) Using the slope of the linear sections of the graph in (c), find the dipole moment of the magnet $p$.

(h) Consider the following thought experiment. You place the dart vertically on the ground, so that it sits on its flights, and you approach it with another dart which has the same magnet at the tip, but with the poles reversed. At some critical distance $h$ between the magnets the first dart will get off the ground and stick to the second dart. Using your experimental results, find $h$.

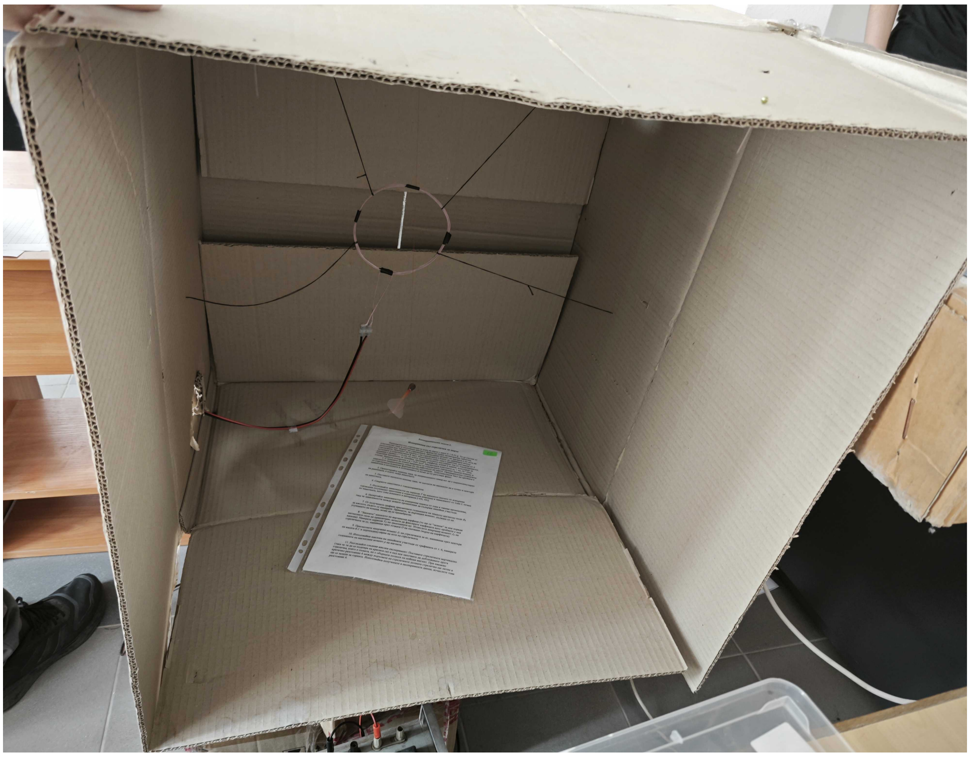


Figure 7

*Each problem is worth 15 points.*
*Time: 5 hours.*